A Theory of Anaphoric Information

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1 Introduction

Work in the semantics of anaphora over the last decade has been dominated by two theoretical approaches: Kamp's Discourse Representation Theory (Kamp and Reyle, 1993) and dynamic logics, of which Dynamic Predicate Logic (Groenendijk and Stokhof, 1991b) is a major example. In this paper, I wish to present a new approach which is based on an explicit modelling of the information shared between an anaphor and its antecedent. The theory – called Theory of Anaphoric Information, or TAI – attempts to extend the empirical insights of DRT and dynamic logics to the more complex phenomena observed in the semantics of plural anaphora, as well as to predict a wider range of readings in the more traditional territory of anaphora under quantification. Both DRT and dynamic logics have addressed these problems, but in each case, there are limitations to the coverage of these theories and predictions which are not in keeping with speakers' intuitions about the linguistic data. A further point addressed by TAI is degree to which the formalism must be dynamic and/or representational, in the sense of the terms used by Groenendijk and Stokhof. TAI represents the information shared between anaphors and their antecedents in a way which resembles the extensions of predicates in the model theory and replaces the dynamic notion of change of information state with something resembling model-theoretic truth conditions.

Two key ideas are context and referent, which are introduced (as they are employed in TAI) in the rest of this section. Section 2 presents the theory as it applies to anaphora involving plural noun phrases. The relevant parts of the logic are described as they are needed, and the complete story can be found in an appendix, with a worked out example of its application. The data we will be interested in include cases where the referent of a plural anaphor is identical to that of its antecedent, and where it is formed by "gathering up" individuals, for example under quantification. The first case is illustrated by

(1) Most farmers own a donkey. They/The farmers are very cruel.

where the anaphor *they/the farmers* refers to the farmers who own a donkey. As an example of the second case, we could continue with

(2) They/The donkeys have a bad time.

in which *they/the donkeys* refers to all the donkeys owned by the farmers, even though the antecedent is a singular NP. Further complications arise when there are anaphors to both antecedents, as in

(3) They beat them.

which typically means that each farmer beats the donkey he (or she) owns. Examples of this last sort require a mechanism for maintaining connections between the information contributed by each antecedent at a finer-grained level than just the plural referents.

The issue of acceptability, which is discussed in section 3, concerns the conditions under which an anaphor, or more precisely the relation between an anaphor and a given antecedent, is acceptable. DRT and dynamic logics address this question in a restricted way through "accessibility" principles, which predict the unacceptability of (some readings of)

(4) Every farmer owns a donkey. ?It is called Chiquita.

What is interesting about this example is that although the antecedent and anaphor agree syntactically – normally taken as a minimal requirement for coreference – the connection between them is still not allowed. We will suggest that several sources of information are needed for a fully adequate account of acceptability, each covering some part of the data, and that the interactions between them are complex.

A final section looks briefly at the methodological issues of compositionality, representationality and the use of dynamic as opposed to static logics.

1.1 Contexts and Referents

The basic idea in TAI is to take each antecedent as contributing information to a *context*, and then use this information to interpret anaphors. Exactly the same can be said of DRT and dynamic logics, and the differences between the theories lie in the nature of the context and how it is used. For TAI, the context contains the *referent* of the antecedents, in the sense of the specific entities which the antecedent was taken to refer to when the sentence containing it was interpreted. We label each such referent with an index, and speak of the part of the context containing it as the *slot* with this index. Anaphors could be looked at as taking information out of an existing slot and using it in their interpretation. A more uniform view is to treat anaphors as also having an associated slot of the context, and to make the link between antecedent and anaphor by expressing a relationship between these slots. The relationship will often be one of equality, but there are cases where other relationships apply. A simple example is "partial full NP anaphors", for which the referent is only part of antecedent's one:

(5) Five couples went to a party. The men [who came to the party] had a good time.

where we take the slot containing the referent of "the people who came to the party" and extract just the men from it.

To turn this into a semantic theory, we must specify the detailed form of the context, and to set it in an appropriate place in the interpretation process. In DRT, the context is formalised as discourse representation structures (DRSs), which are set in place by linking them to the (parsed) discourse and to a model. Similarly, in dynamic logics, the context takes the form of assignment functions, which are derived when interpreting formulae of the logic with respect to a model. TAI is closer to the latter, the main difference (in the abstract) being that its logic is interpreted by two separate interpretation functions. One of them gives truth conditions on the model, while the other gives the conditions constraining the context. By taking this approach, the representation of the context is abstracted out of the semantic mechanism used to obtain it, allowing us to "design" the form of the context purely on the basis of intuitions about what sentences mean, and thus making the theory more strongly led by empirical considerations.

The form of the context is broadly similar to the way predicates are represented in the model. Firstly, it is extensional, consisting purely of the entities making up each referent. Secondly, the context follows much the same ontology that we typically use for predicates in model theoretic semantics, namely a set consisting of tuples of entities. For this reason, the formalisation of the context is called a discourse set (DS). Our goal in what follows will be to define precisely the form of the DS from empirical considerations, and to find a suitable logic for constraining it.

2 TAI and Plural Noun Phrase Anaphora

In this section, we look at the semantics of plural noun phrase anaphora and in doing so present the core of TAI. The semantics proceeds by translating sentences into formulae of the logic L(GQA), and then interpreting them with respect to a model and a discourse set (DS) to give the truth conditions and the conditions on the anaphoric information. The anaphoric conditions are used to define the content and structure of the DS passed on to the next sentence, or (equivalently) to constrain the range of possible DSs consistent with the sentence. The conditions resulting from the interpretation are stated in a meta-language consisting of standard set operators and Link-style sum operators (Link, 1983). A number of additional operators derived from the basic ones are also used. We start by identifying the form DSs must take, and then give some examples of the logic in use.

The structure that we will need for a DS is a set of tuples whose members are Link-style sums of individuals. The referent of an antecedent with index i is found by collecting together the sums found in the ith member of all the tuples in the DS. To keep the distinction between the members of a set and the members of a tuple clear, the latter is referred to as a *slot* in what follows, and the same term is also used to refer to the result of collecting up a given slot from each tuple in the set. The tuples themselves represent the relations between antecedents at a fine level of detail. Tuples may contain a special "empty" element, denoted $\perp$, for reasons to be discussed. The notation $\oplus$ is used for the sum operator, with the minor extension that for any object x, $x \oplus \perp = x$. To give an example DS, in which the first slot in each tuple contains a boy or sum of boys, and the second slot a rose or sum of roses:

$$\{\langle b_1, r_1 \oplus r_2\rangle, \langle b_3, \bot\rangle, \langle b_2 \oplus b_3, r_3 \oplus r_4 \oplus r_5\rangle\}$$

The meaning of this ontology, and the need for it, will emerge in the course of the discussion which follows.

2.1 DSs and data

Perhaps the simplest possible anaphor is a singular antecedent with a singular anaphor, as in

(6) A^i man walks. He$_i$ is happy.

where the superscript indicates the index of the antecedent (identified with a slot in tuples of the DS), and the subscript indicates that the anaphor refers to it. A wide variety of representations would do equally well for this case: all we need is somewhere to store the individual we take *a man* as referring to. The same applies with more than one singular antecedent, as in

(7) A^i man took a^j dog for a walk. He$_i$ threw sticks for it$_j$.

However, once we start to introduce antecedents which refer to more than one individual, more complex structures are needed. In

(8) Every^i farmer in this county owns one^j donkey. They$_i$ beat them$_j$.

the anaphors refer to all of the farmers and all of the donkeys described by the first sentence. Furthermore, the most salient meaning of the second sentence is that each farmer beats the particular donkey he owns. The way we represent this in the DS is by setting up tuples which relate each farmer to the donkey he owns. For example, if the farmers were f_1, f_2, and so on, owning donkeys d_1, d_2, ..., respectively, then the DS would be $\{\langle f_1, d_1\rangle, \langle f_2, d_2\rangle, \ldots\}$, ignoring the contributions of any other antecedents in the discourse[1]. In interpreting *they beat them*, *they* directs us to look at the members of the i slot of the DS and for each such member, we determine the corresponding contents of the j slot; thus for f_1 in slot i, we find d_1 in slot j. We refer to this connection between antecedents, considered at the level of the entities comprising them, as their *dependence relation*.

More complex dependence relations are seen in sentences which receive collective or cumulative readings (Scha, 1984). For example, in

(9) Three^i boys bought five^j roses. They$_i$ took them$_j$ home.

the first sentence can be read as meaning that a total of three boys bought a total of five roses – the cumulative reading – and the second that the roses which were taken home by each boy, or collection of boys, were the ones bought by the boy or boys in question. To get the right dependence relation in the case where boy b_1 bought roses r_1 and r_2, and boys b_2 and b_3 pooled their money to buy the other three, the DS would be $\{\langle b_1, r_1 \oplus r_2\rangle, \langle b_2 \oplus b_3, r_3 \oplus r_4 \oplus r_5\rangle\}$. We cannot talk about the roses which b_2 bought as an individual, since there were no such roses, and so only the sum of the b_2 and b_3 appears in slot i. The interpretation of the second sentence proceeds as before: take each entity in slot i in turn and find the corresponding entity in slot j. So the truth conditions of the second sentence are that b_1 took roses r_1 and r_2 home, and that boys b_2 and b_3 jointly took roses r_3, r_4 and r_5 home.

Evidence that we really do use the dependence relationships established by the antecedent comes from a phenomenon of "cross-sentential counting". Suppose

[1]Here and elsewhere, we will show i in the first member of tuples, j in the second and so on. We could just as well have written 1 for i and 2 for j throughout.

following

(10) Three boys buy five roses.

the discourse continued with

(10a) Joe liked his one.

(10b) Fred ate his one.

(10c) Bill threw his ones/?one away.

Then, (10c) with *one* rather than *ones* is unacceptable. On the model developed here this is because after (10), we can form any context with three boys and five roses partitioned so that they stand in the relation of buying. There might be more than one way of doing so, perhaps because we do not know in that much detail how the boys bought the roses. However, after each of (10a) and (10b), some of the possible contexts have to be discarded, namely ones which have Joe or Fred related to more than one rose, assuming that *one* is interpreted as something like finding the singular part of the slot corresponding to the specified member of another. When we come to interpret (10c), the only contexts remaining have Joe and Fred each linked to one rose, and Bill must therefore be linked to all of the remaining roses, of which there are three. This is not consistent with the singular use of *one* in (10c) and hence it will be rejected, while *ones* will be accepted for similar reasons.

One antecedent can stand in different dependence relationships to two or more others, as in (11), with both *buy* and *own* read collectively:

(11) Three^i boys who bought five^j roses own four^k vases.

Suppose that Joe bought one rose, and Bill and Fred two each, and that Fred and Joe each own one vase and Bill owns two. Then the DS is

$$\{\langle j, r_1, v_1\rangle, \langle f, r_2 \oplus, r_3, v_2\rangle, \langle b, r_4 \oplus r_5, v_3 \oplus v_4\rangle\}$$

Continuing with

(12) They$_i$ put them$_j$ in the$_k$ vases.

an interpretation that each boy put his rose(s) in his vase(s) is possible. Now consider a different situation, where Joe bought one rose, and Bill and Fred jointly bought the other four, and where Fred owns one vase and Joe and Bill own the other three. In this cases (12) is not coherent on a reading where we try to maintain the dependence, paraphrased roughly as *They put their own roses in their own vases.* To represent this, we allow tuples in the DS which contain the special element $\perp$, interpreted as meaning that there is nothing in the given slot corresponding to the other members of the tuple. In the example here, the DS is

$$\{\langle j, r_1, \perp\rangle, \langle b \oplus f, r_1 \oplus r_2 \oplus r_3 \oplus r_4, \perp\rangle, \langle f, \perp, v_1\rangle, \langle j \oplus b, \perp, v_2 \oplus v_3 \oplus v_4\rangle\}$$

Similar multiple dependence relationships can be established by predicates of three or more arguments, such as ditransitive verbs:

(13) Three boys gave five roses to four girls.

The special element $\perp$ is also used in cases where a slot is empty because the antecedent does not refer to anything, as in

(14) Some^i men own no^j donkeys.

in which slot i contains all the men who own donkeys, and slot j contains only $\perp$, representing the fact that there are no donkeys owned by the men.

Anaphors can establish dependence relations between two antecedents that were previously unrelated. (15) is an example:

(15) Three^i boys went to the flower shop. There were five^j roses in the window.

(15a) They$_i$ bought them$_j$.

(15b) They$_i$ took them$_j$ home.

After (15), there is no dependence relation between slots i and j (or, to look at it differently, any dependence relation between them is possible). After (15a), we require that slot i is partitioned into collections of boys which are linked to collections of roses they bought in slot j. (15b) then means that each boy or boys took home the rose or roses they bought, maintaining the dependence established by (15a).

Dependence relations are, to a degree, flexible. Returning to (9),

(9) Three^i boys bought five^j roses. They$_i$ took them$_j$ home.

the most salient reading of the second sentence has each boy (or collection of boys) taking home the roses bought by the boy. However, it is also possible to read it with some other dependence relation, such as all of the boys together taking home the whole collection of roses. If there are further anaphoric references to the boys and the roses, they may respect either the original dependence relation or the new one established by the second sentence.

We now have all the information in the DS which we need to cover the linguistic data: tuples of sums, possibly containing an empty element. Next we examine how to use DSs in interpreting anaphors, and how to obtain the required constraints on them from the interpretation process.

2.2 TAI interpretation

The logic L(GQA) has two interpretation functions, one of which yields the truth conditions, and the other a constraint on possible DSs, called the anaphoric conditions. The truth conditions make reference to the DS only for sentences which contain anaphors. We will show how TAI interpretation works by means of a number of examples in the rest of this section. The full details of the meta-language and the interpretation functions of the logic appear in an appendix. The version of TAI is slightly simplified from that presented in Elworthy (1993) by omitting the conditions used to determine acceptability. Section 3 returns to this point.

Some of the meta-language first needs to be introduced. The symbol W is used to denote an arbitrary DS. If we have a domain of individuals $\mathcal{E}$ in the model, then we will use $\mathcal{D}$ for the closure under the summation operator $\oplus$ of $\mathcal{E}$, and $\mathcal{D}_\perp$ for the union of $\mathcal{D}$ and the set containing the empty element $\perp$. W is then a subset of $\mathcal{D}_\perp \times \mathcal{D}_\perp \times \ldots$. We define two operations on tuples and DSs (and on sets of tuples of sums in general). *Extraction*, written as a single down-arrow, forms a new tuple of the specified slots of a tuple or of a set of tuples:

$$\langle a_1, \ldots, a_n \rangle \downarrow \langle i_1, \ldots, i_k \rangle = \langle a_{i_1}, \ldots, a_{i_k} \rangle \text{ provided for each } i_j,\ 1 \leq i_j \leq n$$

$$W \downarrow \langle i_1, \ldots, i_n \rangle = \{ w \downarrow \langle i_1, \ldots, i_n \rangle \mid w \in W \} \text{ for a set of tuples } W.$$

We will sometimes want to sum the result of extraction, for which a double down-arrow symbol is used, with definition

$$W \Downarrow \langle i_1, \ldots, i_n \rangle = \oplus (W \downarrow \langle i_1, \ldots, i_n \rangle)$$

where $\oplus$ is the Link sum operator, applied to sets, $\oplus\{s_1, s_2, \ldots\} = s_1 \oplus s_2 \oplus \ldots$. When only a single member of a tuple is being extracted, we will use $W \downarrow i$ rather than $W \downarrow \langle i \rangle$ as a minor notational simplification.

The second operation on sets of tuples is *restriction*, and limits W to members containing specific entities in a given slot. The definition is:

$$W \setminus (X, i) = \{w \in W \mid w \downarrow i = X\}$$

For a predicate symbol P, we write P' for the extension of P in the model. $^*S$ is the closure of a set S under summation, and we will also use P^* in place of $^*P'$. The latter form is used in the representation of plurals, so that if P is true of a collection of entities, then the sum closed form P^* includes not only those entities, but also any combinations of them. One point of empirical significance must be made here. The extension in the model, P' is called the *essential extension*, and includes on those individuals which really "go together". Thus, if Joe and Bill jointly lift one table and Fred lifts another one on his own, the essential extension of *lift* is $\{\langle j \oplus b, t_1\rangle, \langle f, t_2\rangle\}$, but does not contain $\langle j \oplus b \oplus f, t_1 \oplus t_2\rangle$, although the latter appears in the *lift**.

To give some examples, using W for the set presented earlier

$$\{\langle b_1, r_1 \oplus r_2\rangle, \langle b_3, \bot\rangle, \langle b_2 \oplus b_3, r_3 \oplus r_4 \oplus r_5\rangle\}$$

then we have

$$W \downarrow 1 = \{b_1, b_3, b_2 \oplus b_3\}$$

$$W \Downarrow 1 = b_1 \oplus b_2 \oplus b_3$$

$$W \Downarrow \langle 1, 2\rangle = \langle b_1 \oplus b_2 \oplus b_3, r_1 \oplus r_2 \oplus r_3 \oplus r_4 \oplus r_5\rangle$$

The sum-closed extension is needed to get the truth conditions of plurals right. For example, given a 1-place predicate holding only of individuals, it would also give us access to all collections of individuals satisfying that predicate. The essential extension is used in determining the dependence relations in the anaphoric information: the principle is that if a predicate relates two NPs, then the dependence relation between their antecedents is a subset of the essential extension. If we view a model as expressing those things which hold true in the world, then the essential extensions reflect the finest degree to which we can divide up those things. There will be a little more to say about this in section 4, on the subject of how such ideas relate to notions of "situation" or "case".

With these basic ideas, we now proceed to a series of examples of TAI in use. The first example shows a simple sentence with a single antecedent, and illustrates where the content of the anaphoric information comes from. In the second example, dependence relationships are introduced. The third example shows how anaphors are interpreted. A final example concerns reflexive pronouns, and show that their interpretation can be carried out in the same way as discourse anaphors, with the DS factoring out.

Example 1 The simple sentence

(16) Most^i donkeys bray.

receives L(GQA) translation

(16a) $(most\ donkey_s)(i)\ bray_s$

which is interpreted to the meta-language expressions shown below[2]. The expression in (16.T) is the truth conditional part, and (16.A) gives the anaphoric conditions.

[2]The subscript 's' just denotes a one place predicate functioning as a set term, i.e. p_s is shorthand for $\hat{x}[p(x)]$.

(16.T) $most'(donkey^*, bray^*)$

(16.A) $W \Downarrow i \in Max_E(donkey^* \cap bray^*)$

In the truth conditions, (16.T), $most'$ functions as in generalised quantifier theory (Barwise and Cooper, 1981), with definition[3] $most'(P, Q) = card(P \cap Q) \geq card(P)/2$. The appendix contains the full definitions for some sample determiners, and we will present them using a similar shorthand in most of the examples that follow. Expanding out, the truth conditions are equivalent to

$$card(donkey' \cap bray') \geq card(donkey')/2$$

so that the sentence is true if the number of donkeys that bray is at least half of the number of donkeys in total.

For the anaphoric part of the interpretation, we introduce an operator $Max_E(S)$, defined as the largest subset of S all of whose members are sum-maximal, i.e. $\{s \in S \mid \forall s' \in S[Crd(s) \geq Crd(s')\}$, where $Crd(s)$ is the number of individuals comprising a sum s. The subscript E indicates that the set must be distributive, so that the for every sum in the set S all of the atoms making it up are also in S. Later, we will also make use of a simpler Max operator which does not have the distributivity condition. The reason for requiring that some sets are distributive is not of great importance to the current discussion; essentially it prevents sentences containing certain determiners from having a collective or cumulative reading. The result of Max or Max_E will be a singleton set in many cases (as it is here) since S is distributive and results from intersecting two sets which are closed under summation. The anaphoric interpretation (16.A) thus means that the content of slot i is the collection of all donkeys that bray. The way the content is divided up is not defined by (16.A), since the sentence contains only a single antecedent, and hence there is no other slot with which to establish a dependence relationship.

[3]In fact, this is a slight simplification; see the appendix.

Example 2 In this example, we show how dependence is established. Taking a variant on an earlier example,

(17) Two^{i} boys buy three^{j} roses.

the L(GQA) translation is

(17a) $(2_{=}\ boy_s)(i)\ \hat{x}[(3_{=}\ rose_s)(j)\ \hat{y}[buy(x,y)]]$

The translation of the numeral determiners in this example, as $2_{=}$ and $3_{=}$, gives an interpretation that there are exactly two boys who between them buy exactly three roses, and that the corresponding two boys and three roses are passed on to anaphors. L(GQA) makes other readings of the sentence available by using two alternative translations of the numeral determiners, both having the truth conditions of "at least n", but differing in their anaphoric properties. If the determiner is applied to expressions translating into set terms P and Q, then one form passes on the intersection of P and Q (as most determiners do), while the other passes on a sum in this intersection with exactly the given cardinality. The latter case is handled by treating the NP as if it had a null determiner with the numeral functioning like an intersective adjective. To show what the translations look like, the sentence *three boys walk* has the following translations:

1. $(3_{=}\ boy_s)(i)\ walk_s$ "Exactly three boys walk."

2. $(3_{\geq}\ boy_s)(i)\ walk_s$ "At least three boys walk."

3. $(\Phi\ (3(boy_s)))(i)\ walk_s$ "There are three boys who walk."

The null determiner Φ has the same semantics as *some*. [4]

[4] No further use is made of alternative readings of numerals in this section. In the version of TAI presented in Elworthy (1993) they are used to obtain a full range of donkey sentences, rather than the single reading predicted by most versions of DRT and dynamic logics.

The truth conditional and anaphoric interpretations are:

(17.T) $\mathit{2}_{=}'(boy^*, {}^*\{X \in \mathcal{D} \mid \mathit{3}_{=}'(rose^*, buy^*(X))\})$

(17.A) $W \Downarrow i \in Max(boy^* \cap {}^*\{X \in \mathcal{D} \mid \mathit{3}_{=}'(rose^*, buy^*(X))\}) \wedge$
$W \downarrow i \subseteq \{X \in \mathcal{D}_\perp \mid W \backslash (X,i) \Downarrow j \in Max(rose^* \cap buy^*(X)) \wedge$
$W \backslash (X,i) \downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid buy_\perp{}'(X,Y)\}\}$

Here, and in later examples, we use "currying" on predicate extensions, so that $buy^*(X)$ denotes $\{Y \in \mathcal{D} \mid buy^*(X,Y)\}$. The "bottomed" predicate $buy_\perp{}'$ is used here to allow tuples containing $\perp$ into the DS, the need for which was demonstrated by examples such as (11).

Using the determiner definitions in the appendix, the truth conditional part can be paraphrased as saying that there are exactly two boys (and no more) which together are an X that buys exactly three roses. Because of the sum closure operators, the truth conditional interpretation encompasses the distributive, collective and cumulative readings. TAI could be formulated to distinguish the readings by having different kinds of set term (i.e. the expressions such as $\hat{x}[\ldots]$) in a straightforward way. The current approach is taken in the interests of simplicity: our concern is anaphora and not how alternative plural readings should be modelled[5].

The anaphoric part of the interpretation runs as follows. The first line says that the content of slot i ($W \Downarrow i$) is a maximal collection of boys who buy three roses. The second line says that for each member X of slot i, the content corresponding to it in slot j ($W \backslash (X,i) \Downarrow j$), i.e. the sum of sums from slot j that are each related to X, is all the roses bought by X. The final line establishes the dependence relation by saying that the objects Y which together make up

[5]For find extensive discussions on the problems of ambiguity and vagueness in plurals see van der Does (1992) and Verkuyl and van der Does (1991).

this content stand in the buy relation to X. The dependence relation is a subset of the essential extension of *buy*, and not the sum-closed version.

The appendix goes through this example in more detail, showing how the interpretations are derived, and how they relate to a DS for an example model. The principles given here extend easily to sentences with ditransitive verbs and relative clauses.

Example 3 The third example shows how anaphors work. The superscripts in this example indicate the source of the anaphoric information, and the subscripts where further anaphoric information is passed to. The presence of both is to allow for partial NP anaphors which pass on only some of the source's information, and for non-prominent readings which change the dependence relation while leaving the overall content the same. For

(18) They$_i^k$ like they$_j^l$.

the L(GQA) formula is

(18a) $(the\ \ Of(i))(k)\ \hat{x}[(the\ \ Of(j))(l)\ \hat{y}[like(x,y)]]$

The important thing to notice about this is the translation of the anaphors, which take the form of the determiner *the* applied to set terms formed with *Of*, which is interpreted as the set of things found in the specified slot of the DS. The full definition of the *Of* operator is given in the appendix: roughly the truth conditional part of its interpretation makes the contents of a slot available as a single sum, and the anaphoric part of the interpretation equates the contents of the input and output slots. The approach of giving pronominal anaphors a complex translation is so that full NP anaphors such as *the men* in (5) can be represented in a similar way, by intersecting the translation of the noun either with the *Of* set term, or with the set term to which *the* applies, depending on whether the anaphor applies the whole of the antecedent or extracts just part of it.

The truth conditional interpretation is

$$\text{(18.T)}\quad \mathit{the}'(W{\downarrow}k,\ {}^{*}\{X \in \mathcal{D} \mid \mathit{the}'(W\backslash(X,k){\Downarrow}l, \mathit{like}^{*}(X))\})$$

The definition of the meta-language determiner the' is that the supremum (largest member) of its first argument is a member of its second one. In this case, the interpretation is equivalent to

$$W{\Downarrow}k \in\ {}^{*}\{X \in \mathcal{D} \mid W\backslash(X,k){\Downarrow}l \in \mathit{like}^{*}(X)\}$$

which says that slot k is a sum of things X such that the contents of slot l corresponding to X appearing in slot k is liked by that X.

For the anaphoric part, we have

$$\begin{aligned}\text{(18.A)}\quad & W{\Downarrow}k \in \mathit{Max}(W{\downarrow}k \cap\ {}^{*}\{X \in \mathcal{D} \mid W\backslash(X,k){\Downarrow}l \in\ \mathit{like}^{*}(X)\}) \\ & \wedge W{\downarrow}i \subseteq \{X \in\ \mathcal{D}_{\perp} \mid W\backslash(X,k){\Downarrow}l \in \mathit{Max}(W\backslash(X,k){\downarrow}l \cap\ \mathit{like}^{*}(X)) \\ & \qquad\qquad \wedge W\backslash(X,k){\downarrow}l \subseteq \mathit{like}_{\perp}{}'(X)\} \\ & \wedge W{\Downarrow}i = W{\Downarrow}k \wedge W{\Downarrow}j = W{\Downarrow}l\end{aligned}$$

Most of this holds trivially when the truth conditions do. The important points are that the dependence between slots k and l is defined by like', just as buy' defined the dependence relation in the previous example, and that the anaphoric information is passed from the antecedent by means of the final two conjuncts. As it stands, this formula requires that the content of the slots is passed on, but puts no restrictions on the structure of the information, in the form of the dependence relations. It therefore corresponds to the non-prominent reading of the sentence. Following (17), it would hold if the boys like different roses to the ones they bought. For the prominent reading, the input and output indices are set equal. The final two conjuncts are now trivially satisfied, while the dependence is maintained because slots i and j (or identically slots k and l) stand in both the buy' and $like'$ relations.

A final (and minor) remark about this example is that the truth conditional part of the interpretation can be interpreted independently of the sentence(s) containing the antecedent, since it refers only to the *output* slots k and l. The only connection between the antecedents and the anaphors in TAI appears in the anaphoric part of the interpretation.

Example 4 As a final example, we show how reflexive pronouns are captured in TAI. For

(19) Every^i cat washes itself^i_i.

the translation and truth conditional interpretations are:

(19a) $(every\ cat_s)(i)\ \hat{x}[(the\ Of(i))(i)\ \hat{y}[wash(x,y)]]$

(19.T) $every'(cat^*,^*\{X \in \mathcal{D} \mid W \backslash (X,i) \Downarrow i \in wash^*(X)\})$

It holds as an identity that $W \backslash (X,i) \Downarrow i = X$, and hence the truth conditional interpretation reduces to

$$every'(cat^*,^*\{X \in \mathcal{D} \mid X \in wash^*(X)\})$$

which does not involve the DS at all, and which means that every cat is an X which washes X. The anaphoric part of the interpretation does not make use of any properties beyond the ones discussed, and sets slot i to be all cats that wash themselves.

2.3 Completing the story

We now fill in a few details which have been skipped over in the description so far. Firstly, to obtain the discourse level of the interpretation, going beyond single sentences, TAI makes use of an extended logic L(GQAD). The syntax is the same as that of L(GQA), with the addition of a sequencing operator which applies to two closed formulae, written as $F_1; F_2$. Sentence level conditionals and modal subordination can also be treated at this level, although we do not go into the details here. Interpretation in L(GQAD) is carried out with respect to a set of DSs, written as V. The interpretation of a closed formula F with respect to V is defined as the largest subset of V each member of which validates both the truth conditional and anaphoric interpretations of F. The sequencing operator can be interpreted equivalently as either the interpretation of F_1 with respect to V intersected with that of F_2; or as the interpretation of F_2 with respect to the set of DSs resulting from interpreting F_1 with respect to V. The latter formulation has a more "dynamic" flavour to it. Either form is consistent with the underlying view that interpreting a discourse can be seen as progressively constraining the possible models and DSs.

A slightly more important issue is that of how we translate from sentences of English to formulae of TAI. The approach taken in Elworthy (1993), which is given in detail in the appendix, is to define a simple (and simple-minded) phrase structure grammar, and to associate a semantic rule with each rule of the grammar. The semantic rules are nothing special; for example the rule for making a sentence from a NP and a VP just applies the translation of the former to that of latter. The rules are fully compositional, in that the translation of a phrase is a simple combination of the translations of its constituents, the whole process being bootstrapped from lexical translations. This much is relatively straightforward; however, we also allow some additions to the simple translation model. Firstly, all NPs have an associated index, which is taken to correspond to the DS slot, with pronouns having two indexes in the manner of the examples above. Any method of choosing unique indices may be used, for example assigning integers from left to right across the discourse, except that it is expected some anaphor resolution procedure will set input indices on anaphoric NPs to the indices of their antecedents. Secondly, we specify *preference principles*, which are intended to capture pragmatic conditions. For the fragment discussed so far, only one preference principle is needed, namely that anaphor output indices are the same as their input indices, thus providing the more prominent dependence-maintaining reading by preference.

For completeness, we should also have a means of making alternative quantifier scopings available, either by something like Cooper storage or by rearranging the translations. However, there is another way of handling quantifier scoping, by expressing them as constraints on DSs, and hence capturing quantifier scope ambiguity entirely in model-theoretic terms. If i is the index of a subject NP and j that of an NP in object position, then we could force the object to have wide scope by requiring that the contents of slot j corresponding to a member of slot i is the same for all such members and is equal to the whole of slot j, i.e.

$$\forall X \in W{\downarrow}i[W\backslash(X,i){\Downarrow}j \neq \bot \rightarrow W\backslash(X,i){\Downarrow}j = W{\Downarrow}j]$$

In the case of

(20) Every farmer owns a donkey.

this condition is a formalisation of the intuitive statement of the object wide-scope reading that “the same donkey is owned by every farmer”. The same principle may be generalised to scopings involving more quantifiers, and to the reverse scope reading of (17), with each boy buying the same roses, in exactly the same way.

2.4 Some comparisons

We are now in a position to compare the treatment of plural anaphora in TAI with those in dynamic logics and in DRT. A more general discussion of the methodological differences between the theories is left for a later section.

A first point of comparison is whether all the information is gathered into a single context, such as the DS in TAI, or whether multiple contexts are used, each containing part of the information. The latter approach is taken in DPLP, the extension of DPL to plurals by van den Berg (1990) in which interpretation is carried out with respect to a set of assignment functions, each mapping a variable to one of the individuals which together form the referent of the noun phrase. Dependence relations are implicitly built into the assignment functions: two individuals stand in a dependence relation if they are in the domain of the same assignment function. Since the assignment functions contain only individuals, there is no way of representing non-distributive examples of the "boys and roses" sort, although it should be possible to reconstruct the theory to deal with such cases. There is more a problem with obtaining readings which do not respect the dependence relation, since this would require operations applying across the set of assignment functions, in order to extract entire plural referents and change how they are partitioned with respect to one another.

Assignment functions do not have a clear ontological status: they are part of the interpretation process, but not part of either the logical language or the model. The only way I can see of having operations across assignment functions is therefore to introduce an operator into the logic, which, when interpreted with respect to a set of assignment functions carries out the operations we want, but it seems very hard to motivate such an operator on anything other than these technical grounds. Certainly, it has no counterpart in the surface form of the sentences, which raises worries about compositionality. In summary, we must accept either the introduction of such an operator, or we must dismiss readings which change dependence, or we must drop any representation of dependence at all.

Turning to DRT's treatment of plurals in Kamp and Reyle (1993, chapter 4), the key idea is to allow discourse referents to stand for non-atomic entities, whose model-theoretic counterparts are Link-style sums. Non-atomic discourse referents can be formed in a variety of ways, including (amongst others) from plural, collective NPs as in

(21) Three boys buy five roses.

and from NPs within the scope of quantification. An example of the latter is

(22) Every man meets a woman.

where a sum could be applied over the singular discourse referent which stands for each man in turn in the quantification, and similarly for the object NP. Kamp and Reyle specifically omit cumulative uses of plurals from their fragment, and hence some of the issue of dependence do not arise. Following (21), reference as in

(23) They paid for them.

involves only knowing the identity of the complete group of boys and the complete group of roses, without any information about how they are subdivided. Dependence does apply in the simpler case of distributive quantification, so that DRT would interpret

(24) They like them.

following (22) as meaning that each man likes the woman he meets. The dependence is maintained by copying the DRS conditions which related the original singular discourse referents for the NPs in (22), over which abstraction was applied, and there is no way of violating the dependence relation, other than by going to a weaker reading where the collection of men as a whole like the collection of women as a whole.

TAI's explicit representation of context also emphasises the distinction between "direct" and "indirect" anaphora. By indirect anaphora, we mean pronouns which refer to something related to an antecedent but not explicitly described by it. An example is

(25) There are 100 defective Hondas in this garage. They are good cars, though.

where *they* refers to Hondas in general, and not just the defective ones in the garage. Indirect anaphora tend to sound less natural, and require some degree of accommodation for their interpretation. DRT's approach to sentences such as

(26) Most farmers own a donkey. They are cruel to them.

(27) Many farmers own a donkey. They beat it.

can be seen as being indirect, relying on accommodation principles to assemble plural discourse referents from singular ones. In contrast, TAI treats the singular and plural anaphora uniformly, by using either the whole of a slot, or the part of it which follows from restriction. Accommodation would still be required for 25 in TAI[6].

Summarising, the key aspects of TAI as compared to DRT and dynamic logics is in the treatment of the context as being explicit and of similar status to the

[6]Thanks to one of the Linguistics and Philosophy reviewers for suggesting this discussion.

model, and in the fine-grained representation of the dependence, particularly between plurals.

3 Acceptability of Anaphors

The final empirical matter is the acceptability of anaphors, or more precisely of the antecedent-anaphor relation. DRT goes some way to addressing this question by defining accessibility for discourse referents. I will suggest here that other information has to be taken into account, and will show how some of this information can be captured in TAI. One caveat in all this is that the data are hard to interpret, and vary considerably with context and from one speaker to another. Roughly speaking, there are three likely approaches, which can be characterised as *local-syntactic*, *structural* and *referential*. In each case, the anaphor possibilities are predicted by the values of features of the anaphor and antecedent, such as number (which we concentrate on here) and gender. The realisation of the features varies between the three approaches.

3.1 Models of acceptability

On a local-syntactic approach, anaphor possibilities are predicted by syntactic features of the anaphor and antecedent which are independent of their position in the overall syntactic structure. The local-syntactic approach is sufficient to predict the unacceptability of

(28) A man walks in the park. ?They are happy.

and similar examples, but falls down on

(29) Every doctor was working late last night. ?He had too many patients.

Here *every doctor* is syntactically singular, and agrees with a singular verb phrase, but singular *he* is still not acceptable.

The account might be saved if we allow NPs to have two number features, one for syntactic agreement and one for anaphoric agreement. However, there are further problems with

(30) Most farmers own a donkey. They are mangy beasts/?It is a mangy beast.

In this example, it is the presence of *most farmers* which licences the plural anaphor and blocks the singular one, even though it does not form part of either the anaphor or the antecedent itself. It therefore appears that the local syntactic principles on their own are not sufficient. A solution is to adopt a structural approach in which either the syntactic structure of the sentence or structural properties of the semantic representation – ones which are independent of the interpretation – are used. DRT is an example of a structural theory, in which the singular anaphor in (30) is disallowed by means of an accessibility relationship stating (roughly) that singular antecedents within a quantification are inaccessible to singular anaphors outside the quantification.

Problems for structural accounts arise when the acceptability can only be predicted with respect to the situation being considered. To see this, consider an example presented by Kamp and Reyle (1993) (in fact, used by Kamp and Reyle to motivate a structural constraint on accessibility in DRT):

(31) Every director gave a present to a child from the orphanage.

(31a) One of them opened it/its present straight away.

(31b) One of them opened ?them/?its presents straight away.

(31c) Two of them opened them/their presents straight away.

Kamp and Reyle claim that (31a) and (31c) are acceptable, but (31b) must be rejected. However, these judgements may be the result of preconceptions about typical situations in the real world, namely that there are fewer directors than there are children. (32) is structurally the same as (31), and yet (32a) seems less good than (31a) and (32b) rather better than (31b):

(32) Every child gave a present to a teacher from the school.

(32a) One of them opened ?it/?its present straight away.

(32b) One of them opened them/its presents straight away.

The preconception this time is that there are more children than teachers. At least one teacher must therefore have received more than one present, and if that teacher is the subject of (32b), then it becomes acceptable. Summarising, the acceptability is contingent on what the model contains. A purely structural approach does not have access to this information, and hence cannot make the right predictions.

The data in (31)–(32) suggest adopting a *referential* account, according to which the acceptability is determined by looking at what the anaphor and antecedent refer to. In (31a) and (32a), the fact that *it* is singular would require that the object referred to by the anaphor was an individual, and similarly in (31b) and (32b), *them* requires a plural referent. TAI takes a referential approach to acceptability, which we illustrate in the next section. It turns out that the details of a referential theory of acceptability are quite hard to get right, and following the presentation of the TAI approach, we will discuss whether the referential account is sufficient on its own. However, we first briefly describe the mechanism used in plural DRT to illustrate some of the complexities required in a structural account.

In Kamp's original version of DRT (Kamp, 1981), accessibility was a relatively simple matter, with certain DRS constructs such as universal quantification and conditionals blocking access to the discourse referents contained in sub-DRSs of those constructs. DRT with plurals (Kamp and Reyle, 1993) introduces a number of additional mechanisms, of which the two principal ones are as follows. Firstly, a distinction is made between atomic and non-atomic discourse referents, captured by a DRS condition of the form *at(x)* or *non-at(x)*. Such conditions do have a model theoretic interpretation, but their major use is in the selection of appropriate antecedents, where (roughly) they are used to restrict the discourse referents accessible to an anaphor. In fact, their role at the level of the model theory is a little odd, since the DRS construction algorithm will ensure that they are always satisfied except in certain degenerate cases such as

(33) Every farmer owns a donkey. They are cruel.

in a model where there is just one farmer. DRT does not separate the falsity of atomic/non-atomic conditions from falsity for "genuine" truth conditional reasons. In the above example, the DRS embedding could fail because there is only one farmer, or because the farmer is not cruel. It is not clear to me that this is really correct: it seems to confuse the representation of properties of the discourse referents and properties of the world as reflected in the model.

The second mechanism Kamp and Reyle use is *pl* annotations on discourse referents. The purpose of such annotations is to indicate that although a discourse referent is not necessarily plural in itself, it is connected with an antecedent which was. Kamp and Reyle's example of this is

(34) Three lawyers hired a secretary they liked.

on a distributive reading, which has *they* referring to a single lawyer drawn from a plural collection. The discourse referent for *they* is marked with *pl* to reflect this, allowing the discourse referent to have a plural antecedent while being atomic. A refinement of *pl* annotation is used for discourse referents which are dependent on the plurality of other discourse referents. One of Kamp and Reyle's major examples is (31), where, for example, the acceptability of *them* in (31c) is predicted by marking it as being plural dependent on *two*. The DRS construction rules for getting atomicity conditions and *pl* marks right become quite complex, and one goal of the referential account is to find a simpler account.

3.2 Referential acceptability in TAI

The referential account is particularly appropriate for TAI, since there is explicit access to the anaphoric information in the form of the discourse set. To specify the conditions which the referent must meet, the translation of anaphors is changed from the form we used earlier, namely $(\mathit{the}\ \mathit{Of}(i))(k)$ to expressions such as $(\mathit{the}\ \mathit{Of}(i,P))(k)$. Here P is called an *agreement predicate*, and may take on the values sg, pl, un, for singular, plural and unconstrained information. The extensions of these predicates are all atoms in the model ($\mathcal{E}$), all non-atomic sums ($\mathcal{D}-\mathcal{E}$), and all entities other than $\perp$ ($\mathcal{D}$), respectively. The truth conditional interpretation of the set term $\mathit{Of}(i)(k,P)$ is no different to that for $\mathit{Of}(i)(k)$. The anaphoric part of the interpretation is changed to add a condition that the contents of slot i is in the extension of P. So, for example, the anaphoric interpretation of

(35) He_i^k walks.

which without the agreement predicate was

$$W \Downarrow k \in \mathit{Max}(W \downarrow k \cap\ \mathit{walk}^*) \wedge W \Downarrow i = W \Downarrow k$$

now becomes

$$W \Downarrow k \in Max(W \downarrow k \cap \; walk^*) \wedge W \Downarrow i = W \Downarrow k \wedge W \Downarrow k \in sg'$$

If the antecedent in slot i is not singular, the anaphoric conditions will come out as false, and we can hence mark (35) as unacceptable.

One further refinement we will introduce is that of agreement under restriction. In this case, rather than the agreement predicate applying to the whole of a slot, it applies to a slot restricted by a value in another slot using the restriction operator introduced earlier and written $W \setminus (X, i)$ for example. Although TAI does not cover partitives at present, this is how (31)–(32) would be dealt with. For example, in (32a),

(32a) One of them opened ?it/?its present straight away.

with i and j being the output slots for *one of them* and *it*, the idea is that if X is a child referred to be *one of them*, then the agreement predicate of *it* applies to $W \setminus (X, i) \Downarrow j$. We may also note that the referential account deals adequately with many of the simpler cases, such as (36)–(38)[7].

(36) A man walks in the park. He whistles/?They whistle.

(37) Some men walk in the park. They whistle/?He whistles.

(38) Many farmers own a donkey. They are mangy beasts/?It is a mangy beast.

In each case, the unacceptable anaphor is rejected because the relevant slot contains an entity of the wrong sum-cardinality. As a final example, the referential acceptability also predicts

[7]Assuming (38) is not being given a modal subordination reading (Roberts, 1987,1989), where the second sentence is taken as effectively lying inside the quantification of the first.

(39) No man owns a donkey. ?He/?They beat it.

with *no* taken non-intersectively, since the slot for the subject NP will be empty, and hence any agreement predicate will fail on it.

So far, the referential account looks promising. However, there are examples which create problems. The first occurs with sentences containing two anaphors, such as

(40) Some boys each buy a rose. They give it to a girl.

Using agreement under restriction, we get the right prediction for *it* as being acceptable. But we can also have dependent plural examples such as

(41) Some boys each buy a rose. They give them to girls.

which can be acceptable in exactly the same circumstances as (40), when each of the boys give the rose he bought to a girl. In this case, the part of the "rose" slot restricted to each boy in turn does not meet the apparent agreement requirements of the plural *them*. One possibility for resolving this problem is to say that singular pronouns obey agreement under restriction and have *sg* for their agreement predicate, while plural pronouns agree with the *whole* slot using *pl* as the agreement predicate. This fixes the problem, although in an inelegant and ad hoc manner.

A more serious problem occurs in the context of donkey sentences. It is well established – see, for example Heim (1990), Schubert and Pelletier (1989) and Chierchia (1992) – that sentences such as (42)

(42) Every^i farmer who owns a^j donkey beats it/?them$_j$.

have a number of different readings, depending on whether the subject NP is taken to refer to farmers who own only one donkey, or to farmers who own donkeys in general, and in the latter case whether the anaphor refers to one, some or all of the donkeys they own. However, regardless of which reading is assumed, *it* is acceptable and *them* is not. The referential account of agreement as outlined above will incorrectly block *it* in the latter two readings and incorrectly allow *them* on all of the readings. A DRT-style structural account has no problems here. The only way I can see of saving the referential account here is to suggest that for each farmer X, the entire set in slot j corresponding to X, i.e. $W \setminus (X, i) \downarrow j$ is made up of singular entities, possibly more than one of them. Unfortunately, we then have to explain why the number test should apply member by member to a set here and to the whole of a slot in order to predict the unacceptability of

(43) Most^i farmers own a donkey. ?He$_i$ beats it.

where $W \downarrow i$ is a set of singular entities following from the distributivity of *most*. In short, while the referential account was introduced because of examples where the acceptability was contingent on the interpretation, (42) is an example where the acceptability must not be made contingent on the interpretation.

As a final source of confusion, note that data and judgements about them are sensitive to many factors. For example, (44), based on an example from Neale (1990), represents only a small syntactic change from the standard donkey sentences, and yet is acceptable (to some speakers, at least) with either singular *or* plural anaphors.

(44) Every farmer who owns more than one donkey beats it/them/all of them.

The data also varies greatly between speakers, so that some people find 45

(45) Every$_i$ farmer owns a donkey. They^i go the market on Thursdays.

fine, while others reject it, presumably because of the (local-)syntactic mismatch between the antecedent and the anaphor, i.e. disregarding both structural and referential information.

A conclusion from this discussion is that a fully adequate account of antecedent-anaphor agreement must draw on more than one kind of information, and that – given the variability in intuitions – it is not possible to specify one source of information as being the definitive one. Furthermore, there is a core of phenomena which can be explained equally by a referential account or by a structural one, and there are data which can only be explained by one of them. Acceptability tends to be a relatively ill-defined issue in semantic theory. There is an acknowledgement that it matters, and some attempts to accommodate it in the theories. What is lacking is a precise model of what acceptability is, and at what level (or levels) to formalise this model. This would be an interesting and fruitful area for future research [8].

4 Methodological remarks

The preceding sections have largely been concerned with empirical examinations of anaphora. We now turn attention to some methodological issues which in part provide the motivation for proposing an alternative to DRT and dynamic logics. The key issues are exactly the ones which led Groenendijk and Stokhof (1987, 1991b) to develop DPL in reaction to Kamp's original version of DRT: to construct a theory which is *compositional* and *non-representational*. The ideal, in the Montagovian tradition, is that the semantic theory is dispensible. It simply provides a means of establishing a relation between sentences and their ultimate interpretation, such as truth conditions with respect to a model. Compositionality ensures each intermediate step, such as the construction of a formula in the logic and its interpretation, draw on only the information present

[8] See also Pollard and Sag (1994) for a broader perspective on factors affecting acceptability and reference.

in the sentences. Non-representationality ensures that once the interpretation has been carried out, then the expressions of the logic may be discarded. If the theory fails to meet either condition, then the interpretation depends on the specific formulation of the intermediate steps between sentences and their interpretation. DRT deviates from compositionality in a number of ways, for example the need to place proper names in the top level DRS wherever they appear in the parse tree, and it is representational in that the embedding of a DRS in a model loses information needed to continue the interpretation, namely the discourse referents and the conditions which define them.

While DPL and other dynamic logics have largely succeeded in overcoming the problems with compositionality, it is not so clear that they fully meet the claims of non-representationality. In particular, when presented with a sentence containing anaphors, it is necessary to know, by name, what variable their antecedents were translated into. The referent of the anaphor is then recovered from this variable and the assignment functions from interpreting previous sentences. Thus, although the formulae of the logic do not need to be retained, variables and assignment functions do, and these are notions of the logic and not of the model. This remains true even in the more recent dynamic logics such as DMG (Groenendijk and Stokhof, 1991a) which separate variables and assignment functions from discourse markers and cases, but still gives them different status to the model.

The approach in TAI has been to push more of the "dynamic" information down into the model, in the form of DSs. DSs do not have exactly the same form as predicate extensions, for example in having an arbitrary number of members in their tuples. However, their underlying ontology is the same in essence, being based on the same notions of set, tuple and sum as the model, without the need to introduce extra categories such as variable. It could be argued that indices do represent an extra category, but this is not so clearly true as it is in the case of variables. They can be seen as a derivative of the notion of tuple, in that if tuples have any meaning different from sets, it comes from being able to distinguish the places in them. A further concern arising from the form that DSs take it that they are somehow too big, in that they contain all information about the referents of anaphors and the relations between them, for all possible anaphors. A topic for further investigation might therefore be to look to a situational account in which the anaphoric information is captured as a collection of infons corresponding roughly to the tuples which appear in DSs, but with the relation that defined the tuple retained; for example each "boys and roses tuple" such as $\langle b_1, r_2 \oplus r_3 \rangle$ might be replaced by an infon (in a very informal way) $\langle b_1\ \texttt{buy}\ r_2, r_3 \rangle$. Note that the infon consists entirely of ground terms[9].

Another important innovation of dynamic logics is to shift the notion of meaning from a truth-conditional point of view to one of change of information state. Interpreting a formula is then not a matter of determining how the model would have to be in order for the formula to be true, but instead expresses how the state of the interpreter changes. However, once the information which is persistent from one formula to another – the anaphoric information in our terms – is expressed in similar terms to the model, these two perspectives on meaning look at lot more similar. The anaphoric interpretation can be seen as successively reducing the set of possible DSs, and the truth-conditional part as doing the same to possible models.

An interesting perspective on these issues has been put forward by Dekker (1994). Dekker proposes a "predicate language with anaphora" (PLA) in which variables and pronouns are treated differently. The referents of pronouns are recorded in information states, which have the same structure as the extension of relations in the model, namely sets of tuples of individuals or *cases*. As the interpretation of the discourse progresses, some of members of the information state will be discarded while others may be extended by bindings for new pronouns. The pronouns themselves are not indexed, and the referent

[9] I am grateful to David Milward for discussion on this point. See also Gawron and Peters (1990).

of a pronoun is extracted from a case using a notion of salience, as reflected in the order that objects appear in the case. Other than this last point, the representation of the anaphoric information is therefore very similar to TAI's discourse sets, in the case of singular anaphora. Dekker's logic is not dynamic, although it draws on some of the ideas that have been explored in this field, such as information update. However, it remains to be seen how will PLA will cope with the problems of plural anaphora, and in particular the need to both maintain dependence relations and allow them to be dropped.

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A Appendix: Formal details of TAI

TAI as presented here excludes the conditional, negation, conjunctions and disjunction operators. A fuller version appears in Elworthy (1993). Some other simplifications have also been applied.

A.1 The meta-language

Basic operators: standard set and truth operators, plus $\oplus, \downarrow, \langle . \rangle$.

Properties and definitions:

1. $\bot$: $\forall x \in \mathcal{E}_\bot[x \oplus \bot = x]$

2. $\oplus$: $a \oplus b = b \oplus a$; $(a \oplus b) \oplus c = a \oplus (b \oplus c)$; $a \oplus a = a$

3. $Ats(a)$ is the set of atoms A such that $\oplus A = a$. $Ats(\bot) = \emptyset$.

4. $Crd(a)$ is the sum-cardinality: $Crd(a) = card(Ats(a))$.

5. $Sup(A)$ is the $a \in A$ such that $\forall a' \in A[(a \neq a') \rightarrow Crd(a') < Crd(a)]$

6. $\langle a_1, \ldots, a_n \rangle \downarrow \langle i_1, \ldots, i_k \rangle = \langle a_{i_1}, \ldots, a_{i_k} \rangle$. Defined only if, for each i_j, $1 \leq i_j \leq n$.

7. $W \downarrow \langle i_1, \ldots, i_n \rangle = \{w \downarrow \langle i_1, \ldots, i_n \rangle \mid w \in W\}$

8. $W \Downarrow \langle i_1, \ldots, i_n \rangle = \oplus(W \downarrow \langle i_1, \ldots, i_n \rangle)$

9. $\oplus\{\langle a_1, \ldots, a_n \rangle, \ldots, \langle z_1, \ldots, z_n \rangle\} =$
$$\langle \oplus\{a_1, \ldots, z_1\}, \ldots, \oplus\{a_n, \ldots, z_n\} \rangle$$

10. $W \setminus (X, i) = \{w \in W \mid w \downarrow i = X\}$

11. ${}^*S = \{Y \in \mathcal{D}^n \mid \exists X \subseteq S[Y = \oplus X]\}$

12. $S(x_1, \ldots, x_n) \ = \ \langle x_1, \ldots, x_n \rangle \in S$

13. $S_\bot(x_1, \ldots, x_n) \ = \ (\langle x_1, \ldots, x_n \rangle \in S) \vee (x_1 = \bot) \vee \ldots \vee (x_n = \bot)$

14. $Distr(S) = \forall s \in S[Ats(s) \subseteq S]$

15. $Max(S) = \begin{cases} \{s \in S \mid \forall t \in S[Crd(t) \leq Crd(s)]\} & \text{if } S \neq \emptyset \\ \{\bot\} & \text{otherwise} \end{cases}$

16. $Max_E(S) = \begin{cases} Max(S) & \text{if } Distr(S) \\ \emptyset & \text{otherwise} \end{cases}$

17. $V \downarrow i = \{W \downarrow i \mid W \in V\}$

A.2 L(GQA)

The syntax (and some aspects of the interpretation) of L(GQA) is based on the language L(GQ) by Barwise and Cooper (1981).

Syntax

Basic expressions:

1. Determiners, written as D below. Examples: *most*, *few*, *every*, *no*, *the*, Φ and for each integer $n \geq 1$, the symbols $n_=$ and $n_\geq$.
2. Variables: $x, y, \ldots$.
3. Predicate symbols: R. Examples: *buy*, *walk*, *farmer*, *own*, *beat*, *john*, *mary*. Each predicate has a specified number of argument places.
4. Indices: i, j.
5. Number terms: n, where n is a natural number.

Derived expressions:

1. Quantifiers: Q.
2. Set terms: S.
3. Formulae: F.

Formation rules:

F1. If Q is a quantifier, i is an index and S is a set term, $Q(i)S$ is a formula.

F2. If R is an n-place predicate symbol, and $x_1, \ldots, x_n$ are variables, $R(x_1, \ldots, x_n)$ is a formula ($n \geq 2$).

F3. If D is a determiner and S a set term, DS is a quantifier.

F4. If x is a variable and F is a formula, $\hat{x}[F]$ is a set term.

F5. If R is a predicate symbol, R_s is a set term.

F6. If S_1 and S_2 are set terms, $S_1 \bigwedge S_2$ is a set term.

F7. If n in a natural number and S is a set term, $n(S)$ is a set term.

F8. If j is an index, $Of(j)$ is a set term.

Rule F6 is used for relative clauses, F7 for some uses of numerals, and F8 for anaphors. A different formulation of the *Of* set term in F8 is used in Elworthy (1993), to include agreement predicates for referential acceptability.

Interpretation

Top-level interpretation:

$[\![F]\!]^{W} = [\![F]\!]_t{}^{W,[\,]} \wedge [\![F]\!]_a{}^{W,[\,]}$

I.e., the overall interpretation is the conjunction of the truth-conditional and anaphoric parts.

Truth conditional part. Points to note are the clause for set terms ($\hat{x}[F]$), which places the binding for the meta-language variable and the slot into the map m. m is a formal device: it does not play the role that assignments or cases do in dynamic logics. The λs combine interpretations and pass indexes from on expression to another, for example from a quantifier arising from a NP to the nominal denotation within it.

$[\![Q(i)S]\!]_t{}^{W,m} = [\![S]\!]_t{}^{W,m} i \in [\![Q]\!]_t{}^{W,m} i$

$[\![R(x_1,\ldots,x_n)]\!]_t{}^{W,m} = [\![R]\!]_t{}^{W,m}(m_1(x_1),\ldots,m_1(x_n))$

$[\![DS]\!]_t{}^{W,m} = \lambda i.([\![D]\!]_t{}^{W,m} i)([\![S]\!]_t{}^{W,m} i)$

$[\![\hat{x}[F]]\!]_t{}^{W,m} = \lambda i.{}^{*}\{X \in \mathcal{D} \mid [\![F]\!]_t{}^{W,m[x/\langle X,i\rangle]}\}$

$[\![R_s]\!]_t{}^{W,m} = \lambda i.[\![R]\!]_t{}^{W,m}$

$[\![S_1 \bigwedge S_2]\!]_t{}^{W,m} = \lambda i.([\![S_1]\!]_t{}^{W,m} i) \cap ([\![S_2]\!]_t{}^{W,m} i)$

$[\![n(S)]\!]_t{}^{W,m} = \lambda i.n' \cap ([\![S]\!]_t{}^{W,m} i)$

$[\![R]\!]_t{}^{W,m} = R^{*}$

$[\![D]\!]_t{}^{W,m} = \lambda i \lambda p.\{X \subseteq \mathcal{D} \mid D'(p,X)\}$, where D' is the determiner definition.

$[\![Of(j)]\!]_t{}^{W,m} = \lambda i.res(W,m) \Downarrow i$

Anaphoric part. The clause for set terms uses an auxiliary function *res* defined below, which converts the bindings in the current map into restrictions on the DS. Effectively it tells us what variables were found in what slots in set terms enclosing the given one. The clauses for determiners take the truth conditions of the set terms they apply to and use them to determine the content, and the anaphoric conditions of the set terms, which constrain the structure of the DS.

$$[\![Q(i)S]\!]_a{}^{W,m} = [\![Q]\!]_a{}^{W,m} i([\![S]\!]_t{}^{W,m} i)([\![S]\!]_a{}^{W,m} i)$$

$$[\![R(x_1,\ldots,x_n)]\!]_a{}^{W,m} = [\![R]\!]_a{}^{W,m}(m_1(x_1),\ldots,m_1(x_n))$$

$$[\![DS]\!]_a{}^{W,m} = \lambda i\lambda t\lambda a.[\![D]\!]_a{}^{W,m} i\ ([\![S]\!]_t{}^{W,m} i)\ ([\![S]\!]_a{}^{W,m} i)\ t\ a$$

$$[\![\hat{x}[F]]\!]_a{}^{W,m} = \lambda i.res(W,m){\downarrow}i \subseteq \{X \in \mathcal{D}_\perp \mid [\![F]\!]_a{}^{W,m[x/\langle X,i\rangle]}\}$$

$$[\![R_s]\!]_a{}^{W,m} = \lambda i.true$$

$$[\![S_1 \bigwedge S_2]\!]_a{}^{W,m} = \lambda i.([\![S_1]\!]_a{}^{W,m} i) \wedge ([\![S_2]\!]_a{}^{W,m} i)$$

$$[\![n(S)]\!]_a{}^{W,m} = \lambda i.[\![S]\!]_a{}^{W,m} i$$

$$[\![R]\!]_a{}^{W,m} = R_\perp{}'$$

$$[\![D]\!]_a{}^{W,m} = \lambda i\lambda t_1\lambda a_1\lambda t_2\lambda a_2.res(W,m){\Downarrow}i \in Max_E(t_1 \cap t_2) \wedge a_1 \wedge a_2$$

if D is a distributive determiner

$$[\![D]\!]_a{}^{W,m} = \lambda i\lambda t_1\lambda a_1\lambda t_2\lambda a_2.res(W,m) \Downarrow i \in Max(t_1 \cap t_2) \wedge a_1 \wedge a_2$$

otherwise.

$$[\![Of(j)]\!]_a{}^{W,m} = \lambda i.W{\Downarrow}i = W{\Downarrow}j$$

Auxiliary function:

$$res(W,m[x/\langle X,i\rangle]) = res(W\backslash(X,i),m)$$

$$res(W,[\ ]) = W$$

Determiner definitions, $D'(p,q)$:

Distributive determiners, applied to $card(p \cap \mathcal{E})$ and $card(p \cap q \cap \mathcal{E})$.

every: $\lambda mn.m{=}n$

most: $\lambda mn.n \geq m/2$

few: $\lambda mn.n \leq f(m)$, for some measure of "fewness" f.

some: $\lambda mn.n \geq 1$

no: $\lambda mn.n = 0$

Other determiners and equivalent alternative definitions.

$$some'(p,q) \;=\; p \cap q \neq \emptyset$$

$$no'(p,q) \;=\; p \cap q = \emptyset$$

$$all'(p,q) \;=\; Sup(p) \in q$$

$$n'_{\geq}(p,q) \;=\; \exists x \in p \cap q[Crd(x) \geq n]$$

$$n'_{=}(p,q) \;=\; \exists x \in Max(p \cap q)[Crd(x) = n]$$

$$\Phi'(p,q) \;=\; p \cap q \neq \emptyset$$

$$the'(p,q) \;=\; Sup(p) \in q$$

A.3 L(GQAD)

L(GQAD) is the discourse-level part of the logic, operating on closed formulae derived from sentences. The version in Elworthy (1993) also includes conditionals and modal subordination.

Syntax

Basic expressions. As L(GQA).

Derived expressions. As L(GQA).

Formation rules. As L(GQA) plus:

D0. If F is a closed formula of L(GQA), then F is a formula of L(GQAD).

D1. If F_1 and F_2 are formulae, then $F_1; F_2$ is a formula (sequencing).

Interpretation

$$I(F,V) \;=\; \{W \in V \mid [\![F]\!]^W\}$$

Sequencing. Several alternative, largely equivalent, definitions are possible

1. $I(F_1;F_2,V) \;=\; I(F_1,V) \cap I(F_2,V)$
2. $I(F_1;F_2,V) \;=\; I(F_2,I(F_1,V))$
3. $I(F_1;F_2,V) = \begin{cases} I(F_1,V) \cap I(F_2,V) & \text{if } I(F_1,V) \neq \emptyset \\ I(F_2,V) & \text{if } I(F_1,V) = \emptyset \end{cases}$

A.4 Translation procedure

The sentence and discourse grammars are combined here. The grammar is simplistic – it is intended to show one way of getting from the surface form of sentence to L(GQA) and L(GQAD) expressions.

Syntactic categories

1. S (sentence)
2. NP (noun phrase), N′ (nominal), N (noun), PN (proper noun), Pro (pronoun)
3. VP (verb phrase), IV (intransitive verb), TV (transitive verb), DV (ditransitive verb)
4. Rel (relative pronoun)
5. Num (numeral)
6. D (discourse).

Production rules

S1. S:qs $\rightarrow$ NP:q VP:s

S2. NP:$p(i)$ $\rightarrow$ PN:p

S3. NP:$p(i)$ $\rightarrow$ Pro:p

S4. NP:$(dp)(i)$ $\rightarrow$ Det:d N′:p

S5. NP:$\lambda s_1.q(s_2 \bigwedge s_1)$ $\rightarrow$ NP:q Rel VP:s_2

S6. VP:p_s $\rightarrow$ IV:p

S7. VP:$\hat{x}[q\ \hat{y}[p(x,y)]]$ $\rightarrow$ TV:p NP:q

S8. VP:$\hat{x}[q_1\ \hat{y}[q_2\ \hat{z}[p(x,y,z)]]]$ $\rightarrow$ DV:p NP:q_1 NP:q_2

S9. N$'$:$s_1 \bigwedge s_2$ $\rightarrow$ N$'$:s_1 Rel VP:s_2

S10. N$'$:p_s $\rightarrow$ N:p

S11. NP:(Φ $n(p)$) $\rightarrow$ Num:n N$'$:p (numerals)

S12. D:f $\rightarrow$ S:f

S13. D:$f_1; f_2$ $\rightarrow$ D:f_1 S:f_2

Sample lexical entries and translations.

N. `boy, boys`: *boy*.

PN. `John`: *john*.

Pro. `it`: (*the* $Of(i)$)

IV. `walks, walk`: *walk*

TV. `owns, own`: *own*

DV. `gives, give`: *give*

Det. `every`: *every*; `most`: *most*; `a`: $\lambda s.(\Phi\ 1(s))$. For numeral determiners, see text.

Rel. `who`.

Num. `one, two, ...`: *1*, *2*. Also see determiners.

A.5 Additional rules

Preference rule 1 Assign anaphor translations the same input and output indices by preference.

A.6 A worked example

As a worked example, here is how

(46) Two^i boys buy three^j roses.

constrains the DS. The sentence is parsed as

(47) $[[\text{Two}^i \text{ boys}]_{NP}\ [\text{buy } [\text{three}^j \text{ roses}]_{NP}]_{VP}]_S$.

and hence is translated to the L(GQA) formula, treating both determiners as meaning "exactly n": $(2_= \ boy_s)(i)\ \hat{x}[(3_= \ rose_s)(j)\ \hat{y}[buy(x,y)]]$

The truth conditional interpretation proceeds in the following manner:

$[\![2_=]\!]_t{}^{W,m} \ = \ \lambda i.\lambda p.\{P \subseteq \mathcal{D} \mid \mathit{2}_='(p,P)\}$

$[\![boy_s]\!]_t{}^{W,m} \ = \ boy^*$

$[\![2_= \ boy_s]\!]_t{}^{W,m} \ = \ \lambda i.\{P \subseteq \mathcal{D} \mid \mathit{2}_='(boy^*,P)\}$

$[\![3_= \ rose_s]\!]_t{}^{W,m} \ = \ \lambda j.\{Q \subseteq \mathcal{D} \mid \mathit{3}_='(rose^*,Q)\}$, similarly

$[\![buy(x,y)]\!]_t{}^{W,m} \ = \ buy^*(m(x),m(y))$ $[\![\hat{y}[buy(x,y)]]\!]_t{}^{W,m} \ = \ \lambda j.{}^*\{Y \in \mathcal{D} \mid buy^*(m(x),Y)\}$

$[\![(3_= \ rose_s)(j)\ \hat{y}[buy(x,y)]]\!]_t{}^{W,m} \ =$

$\quad {}^*\{Y \in \mathcal{D} \mid buy^*(m(x),Y)\} \in \{Q \subseteq \mathcal{D} \mid \mathit{3}_='(rose^*,Q)\}$

$[\![\hat{x}[(3_= \ rose_s)(j)\ \hat{y}[buy(x,y)]]]\!]_t{}^{W,m} \ =$

$\quad \lambda i.{}^*\{X \in \mathcal{D} \mid {}^*\{Y \in \mathcal{D} \mid buy^*(m(x),Y)\} \in$

$\qquad \{Q \subseteq \mathcal{D} \mid \mathit{3}_='(rose^*,Q)\}\}$

Hence, for the formula as a whole:

${}^*\{X \in \mathcal{D} \mid {}^*\{Y \in \mathcal{D} \mid buy^*(X,Y)\} \in \{Q \subseteq \mathcal{D} \mid \mathit{3}_='(rose^*,Q)\}\} \in$

$\quad \{P \subseteq \mathcal{D} \mid \mathit{2}_='(boy^*,P)\}$

Simplifying:

$\mathit{2}_='(boy^*, {}^*\{X \in \mathcal{D} \mid {}^*\{Y \in \mathcal{D} \mid buy^*(X,Y)\} \in \{Q \subseteq \mathcal{D} \mid \mathit{3}_='(rose^*,Q)\}\})$

And finally:

$\mathit{2}_='(boy^*, {}^*\{X \in \mathcal{D} \mid \mathit{3}_='(rose^*, buy^*(X))\})$

For the anaphoric part, the interpretation unfolds as follows:

$[\![2_=]\!]_a{}^{W,m} \ = \ \lambda i.\lambda t_1.\lambda a_1.\lambda t_2.\lambda a_2.res(W,m)\Downarrow i \in Max(t_1 \cap t_2) \wedge a_1 \wedge a_2$

$[\![boy_s]\!]_a{}^{W,m} \ = \ \lambda i.true$

$[\![2_= \ boy_s]\!]_a{}^{W,m} \ = \ \lambda i.\lambda t.\lambda a(res(W,m)\Downarrow i \in Max(boy^* \cap t) \wedge true \wedge a)$

$[\![3_= \ rose_s]\!]_a{}^{W,m} \ = \ \lambda j.\lambda u.\lambda b(res(W,m)\Downarrow j \in Max(rose^* \cap u) \wedge true \wedge b)$, similarly

$[\![buy(x,y)]\!]_a{}^{W,m} \ = \ buy_\perp{}'(m(x),m(y))$

$[\![\hat{y}[buy(x,y)]]\!]_a{}^{W,m} \ = \ \lambda j.res(W,m)\downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid buy_\perp{}'(m(x),Y)\}$

$$[\![(3_= \ rose_s)(j) \ \hat{y}[buy(x,y)]]\!]_a{}^{W,m} =$$

$$res(W,m)\Downarrow j \in Max(rose^* \cap \ ^*\{Y \in \mathcal{D} \mid buy^*(m(x),Y)\}) \wedge$$

$$res(W,m)\downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid {buy_\perp}'(m(x),Y)\}$$

$$[\![\hat{x}[(3_= \ rose_s)(j) \ \hat{y}[buy(x,y)]]]\!]_a{}^{W,m} =$$

$$\lambda i.res(W,m)\downarrow i \subseteq \{X \in \mathcal{D}_\perp \mid res(W,m1)\Downarrow j \in Max(rose^* \cap$$

$$^*\{Y \in \mathcal{D} \mid buy^*(m(x),Y)\}) \wedge$$

$$res(W,m1)\downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid {buy_\perp}'(m(x),Y)\}, \text{where m1} = [x/\langle X,i\rangle].$$

Hence, for the formula as a whole:

$$W\Downarrow i \in Max(boy^* \cap \ ^*\{X \in \mathcal{D} \mid \mathit{3}_='(rose^*, buy^*(X))\}\})$$

$$\wedge W\downarrow i \subseteq \{X \in \mathcal{D}_\perp \mid W\backslash(X,i)\Downarrow j \in Max(rose^* \cap \ buy^*(X)$$

$$\wedge W\backslash(W,i)\downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid {buy_\perp}'(X,Y)\}$$

Simplifying:

$$W\Downarrow i \in Max(boy^* \cap \ ^*\{X \in \mathcal{D} \mid \mathit{3}_='(rose^*, buy^*(X))\}\})$$

$$\wedge W\downarrow i \subseteq \{X \in \mathcal{D}_\perp \mid W\backslash(X,i)\Downarrow j \in Max(rose^* \cap \ buy^*(X))$$

$$\wedge W\backslash(W,i)\downarrow j \subseteq \{Y \in \mathcal{D}_\perp \mid {buy_\perp}'(X,Y)\}$$

Suppose the model is as follows: b_1, b_2 and b_3 are boys, c is some individual which is not a boy, and r_1, r_2, r_3 and r_4 are roses. b_1 buys r_1; b_2 buys both r_2 and r_3 together; c buys r_4. The basic extensions are:

$$boy' = \{b_1, b_2, b_3\}$$

$$rose' = \{r_1, r_2, r_3, r_4\}$$

$$buy' = \{\langle b_1, r_1\rangle, \langle b_2, r_2 \oplus r_3\rangle, \langle c, r_4\rangle\}$$

The extensions closed under summation are:

$$boy^* = \{b_1, b_2, b_3, b_1 \oplus b_2, b_1 \oplus b_3, b_2 \oplus b_3, b_1 \oplus b_2 \oplus b_3\}$$

$$\begin{aligned} rose^* = \{&r_1, r_2, r_3, r_4, r_1 \oplus r_2, r_1 \oplus r_3, r_1 \oplus r_4, r_2 \oplus r_3, r_2 \oplus r_4, r_3 \oplus r_4, r_1 \oplus r_2 \oplus r_3, \\ &r_1 \oplus r_2 \oplus r_4, r_2 \oplus r_3 \oplus r_4, r_1 \oplus r_2 \oplus r_3 \oplus r_4\} \end{aligned}$$

$$\begin{aligned} buy^* = \{&\langle b_1, r_1\rangle, \langle b_2, r_2 \oplus r_3\rangle, \langle c, r_4\rangle, \langle b_1 \oplus b_2, r_1 \oplus r_2 \oplus r_3\rangle, \langle b_1 \oplus c, r_1 \oplus r_4\rangle, \\ &\langle b_2 \oplus c, r_2 \oplus r_3 \oplus r_4\rangle, \langle b_1 \oplus b_2 \oplus c, r_1 \oplus r_2 \oplus r_3 \oplus r_4\rangle\} \end{aligned}$$

Consequently, there are two values of X for which $3'_{=}(rose^*, buy^*(X))$ holds, namely $b_1 \oplus b_2$ and $b_2 \oplus c$. Hence we have

$$^*\{X \in \mathcal{D} \mid 3'_{=}(rose^*, buy^*(X))\} \;=\; ^*\{b_1 \oplus b_2, b_2 \oplus c\} \;=\; \{b_1 \oplus b_2, b_2 \oplus c, b_1 \oplus b_2 \oplus c\}$$

The truth condition (17.T) holds, since the intersection of this set with boy^* is $\{b_1 \oplus b_2\}$, the sole member of which, $b_1 \oplus b_2$, has sum-cardinality of two, in keeping with the determiner definition. Note that this is the cumulative reading.

For the anaphoric part, we have that

$$Max(boy^* \cap {}^*\{X \in \mathcal{D}_\perp \mid 3'_{=}(rose^*, buy^*(X))\}) \;=\; Max(\{b_1 \oplus b_2\}) \;=\; \{b_1 \oplus b_2\}$$

and hence the content of slot i is $b_1 \oplus b_2$. This sum is equal to $\oplus\{\perp, b_1, b_2, b_1 \oplus b_2\}$, and each of these is therefore a possible member of $W \downarrow i$. The effect of substituting each in turn for X in the second line of the anaphoric interpretation is as follows:

1. $X = \perp$: $W \backslash (X, i) \Downarrow j = \perp$
2. $X = b_1$: $W \backslash (X, i) \Downarrow j = r_1$.
3. $X = b_2$: $W \backslash (X, i) \Downarrow j = r_2 \oplus r_3$.
4. $X = b_1 \oplus b_2$: $W \backslash (X, i) \Downarrow j = r_1 \oplus r_2 \oplus r_3$

Finally, the condition in the third line allows the DS to contain $\langle \perp, \perp \rangle$, $\langle b_1, r_1 \rangle$ and $\langle b_2, r_2 \oplus r_3 \rangle$, the former from the definition of $buy_\perp{}'$, and the latter from the essential extension itself. The DS may not contain $\langle b_1 \oplus b_2, r_1 \oplus r_2 \oplus r_3 \rangle$, since it is not in the essential extension. This provides the correct structuring information.

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